For $\alpha=\beta=\frac{1}{6}\pi$, the sides $d\cos\alpha$, $d\cos\beta$ are equal, and the isosceles triangle is equilateral, having its vertex angle equal to $\frac{1}{3}\pi$.

Also solved by H. C. Feemster, Wilmer Thompson, G. B. M. Zerr, and C. N. Schmall.

285. Proposed by C. N. SCHMALL, 604 East 5th Street, New York City.

If R_1 and R_2 are the radii of curvature of an ellipse at the extremities of a pair of conjugate diameters, show that $R_1^{2/3}+R_2^{2/3}=\dfrac{a^2+b^2}{(ab)^{2/3}}$, where a, b, are the semi-axes.

I. Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Let $x^2/a^2+y^2/b^2=1$ be the ellipse. Also let $x=a\cos\theta$, $y=b\sin\theta$.

$$dx=-a\sin\theta\, d\theta,\quad d^2x=-a\cos\theta\, d\theta^2-a\sin\theta\, d^2\theta,$$
$$dy=b\cos\theta\, d\theta,\quad d^2y=-b\sin\theta\, d\theta^2+b\cos\theta\, d^2\theta.$$

$$\therefore R_1=\frac{(dx^2+dy^2)^{\frac{3}{2}}}{dxd^2y-dy\,d^2x}=\frac{(a^2\sin^2\theta+b^2\cos^2\theta)^{\frac{3}{2}}}{ab}.$$

$$R_2=\frac{(a^2\sin^2\phi+b^2\cos^2\phi)^{\frac{3}{2}}}{ab}.\quad \text{But } \phi=\theta+\tfrac{1}{2}\pi.$$

$$\therefore R_2=\frac{(a^2\cos^2\theta+b^2\sin^2\theta)^{\frac{3}{2}}}{ab}.$$

$$R_1^{2/3}+R_2^{2/3}=\frac{(a^2+b^2)\,(\sin^2\theta+\cos^2\theta)}{(ab)^{2/3}}=\frac{a^2+b^2}{(ab)^{2/3}}.$$

II. Solution by HOWARD C. FEEMSTER, Professor of Mathematics, York College, York, Neb.; S. G. BARTON, Professor of Mathematics, Clarkson School of Technology, Potsdam, N. Y., and J. SCHEFFER, Hagerstown, Md.

Let x_1, y_1 be the point of intersection of the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$, with a diameter, and $-\dfrac{ay_1}{b}$, $\dfrac{bx_1}{a}$, the point of intersection of the ellipse with the diameter conjugate to the first.

But the radius of curvature of any point on a curve equals:

$$(1)\quad \rho=\frac{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\dfrac{d^2y}{dx^2}}.$$

For the ellipse,

$$(2)\ \frac{dy}{dx}=-\frac{b^2x}{a^2y},\ \text{and}\ (3)\ \frac{d^2y}{dx^2}=-\frac{b^4}{a^2\,y^3}.$$

From (1), (2), and (3), making the sign positive,

$$\rho=\frac{(a^4y^2+b^4x^2)^{\frac{3}{2}}}{a^4b^4}.$$

Hence, for $(x_1,\ y_1)$, $\rho=R_1=\dfrac{(a^4y_1^2+b^4x_1^2)^{\frac{3}{2}}}{a^4b^4}$.

And for $-\dfrac{ay_1}{b}$, $\dfrac{bx_1}{a}$, $\rho=R_2=\dfrac{(a^2b^2x_1^2+a^2b^2y_1^2)^{\frac{3}{2}}}{a^4b^4}$.

$$\therefore\ R_1^{\frac{2}{3}}+R_2^{\frac{2}{3}}=\frac{(a^2+b^2)\ (b^2x_1^2+a^2y_1^2)}{(ab)^{\frac{8}{3}}}=\frac{a^2+b^2}{(ab)^{\frac{2}{3}}}.$$

MECHANICS.

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235. Proposed by W. J. GREENSTREET, M. A., Marling School, Stroud, England.

A uniform heavy rod turns freely round a hinge at one end and rests with the other against a rough vertical wall, at angle, α, to the wall. Find the angle of arc on which this end may rest, and the pressures at the ends of the arc.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Let $2a$=length of rod; w, its weight; O, the hinge; OBC, the plane of the rod (vertical) perpendicular to the wall; α=angle BOC; μ=coefficient of friction between rod and wall; OA, the position of the rod at any time; OD, the projection of OA on the plane OBC; θ=angle DOA; ϕ=angle DOC; R= reaction or pressure at wall; OA, the axis of x; z, normal to AOD; y perpendicular to both x and z.

Then $OD=2a\cos\theta$, $OC=2a\sin\alpha$, $\phi=\cos^{-1}(\sin\alpha/\sin\theta)$.

The weight of the rod acting at its mid-point is equivalent to $w\cos\phi$ parallel to z, and $w\sin\phi$ parallel to OD. R is perpendicular to both OA and AD. Taking moments about y and z, respectively, we have, $aw\cos\phi=2aR$, $aw\sin\phi\sin\theta=2a\mu R$.

$\therefore \sin\theta=\mu\cot\theta=\mu\sin\alpha/\surd(\cos^2\theta-\sin^2\alpha)$.

$\therefore \sin^2\theta=\frac{1}{2}[\cos^2\alpha\pm\surd(\cos^4\alpha-4\mu^2\sin^2\alpha)]=P$. Use positive sign.

$R=(w/2)\cos\phi=[w\sin\alpha]/[2\surd(1-P)]$.

236. Proposed by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

A simple beam length $2a$, supported at the ends, is loaded with c pounds per running foot at the ends and increases uniformly to the center, where it is b pounds per running foot. Find deflection at center due to this load.